03/11/20 Properties of Logarithms

Remember that logarithms are inverses of exponential functions. Just like we had exponent properties, we have properties for logarithms. You will notice they are very similar…..

PRODUCT PROPERTY of logarithms
$\log_b(mn) = \log_b m + \log_b n$

QUOTIENT PROPERTY of logarithms
$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

POWER PROPERTY of logarithms
$\log_b(m^n) = n \cdot \log_b m$

Here are some other properties that will help you with logarithms:

Inverse properties:

*** $\log_b 1 = 0$ *** $\log_b b = 1$ *** $\log_b b^n = n$ *** $b^{\log_b n} = n$

ex: $\log_2 1 =$ $2^x = 1$ $x = 0$!

ex: $\log_2 2 =$ $2^x = 2^1$ $x = 1$

ex: $\log_2 2^3 =$ $2^x = 2^3$ $x = 3$

These properties can be used to expand or condense logarithmic expressions. You will see this in the following examples!!

$\boxed{\text{Expanding}}$ expressions using the properties of logarithms:

Ex 1: $\log_2(ab^4)^3$

$= \log_2(a^3 b^{12})$

$= \log_2 a^3 + \log_2 b^{12}$

$= \boxed{3\log_2 a + 12\log_2 b}$

Ex 2: $\log_5 \frac{x}{7}$

*Use the quotient property

$\boxed{\log_5 x - \log_5 7}$

Ex 3: $\log_3 \frac{x^2\sqrt{y}}{z^5}$ - Quotient Property!

$\log_3 x^2\sqrt{y} - \log_3 z^5$

$\log_3 x^2 + \log_3 y^{1/2} - \log_3 z^5$

$= \boxed{2\log_3 x + \frac{1}{2}\log_3 y - 5\log_3 z}$

Ex 4: $\log_3 \frac{81\sqrt[4]{a}}{b^2}$ - Quotient Property

$= \log_3 81a^{1/4} - \log_3 b^2$

$= \log_3 81 + \log_3 a^{1/4} - \log_3 b^2$

$= \log_3 3^4 + \frac{1}{4}\log_3 a - 2\log_3 b$

$= \boxed{4 + \frac{1}{4}\log_3 a - 2\log_3 b}$

Ex 5: $\log\left(1000x^3\sqrt{y^5}\right)$ - Product Property

$= \log 1000 + \log x^3 + \log\sqrt{y^5}$

$= \log 10^3 + 3\log x + \log y^{5/2}$

$= \boxed{3 + 3\log x + \frac{5}{2}\log y}$

Ex 6: $\log_2(64xy)$ - Product property!

$= \log_2 64 + \log_2 x + \log_2 y$

$= \log_2 2^6 + \log_2 x + \log_2 y$

$= \boxed{6 + \log_2 x + \log_2 y}$

Condensing expressions using the properties of logarithms:

Ex 7: $\log_2 14 - \log_2 7$

*Subtraction becomes division.

$= \log_2 \frac{14}{7}$ simplify!

$= \log_2 2$

$= \boxed{1}$!

Ex 8: $6\log_7 m - \frac{3}{2}\log_7 n$

*Power property!

$= \log_7 m^6 - \log_7 n^{3/2}$

*Quotient property

$= \log_7 \frac{m^6}{n^{3/2}}$ rewrite!

$= \boxed{\log_7 \frac{m^6}{\sqrt{n^3}}}$

Ex 9: $\log_4 x + 2\log_4 y$

*Power property

$= \log_4 x + \log_4 y^2$

*Product property

$= \boxed{\log_4 (xy^2)}$

Ex 10: $2\log_8 x + \frac{1}{2}\log_8 (x+4)$

*Power

$= \log_8 x^2 + \log_8 (x+4)^{1/2}$

*Product

$= \log_8 x^2 (x+4)^{1/2}$ rewrite!

$= \boxed{\log_8 x^2 \sqrt{x+4}}$

Ex 11: $4\log_6 (x+2) - 3\log_6 (x-5)$

*Power property

$= \log_6 (x+2)^4 - \log_6 (x-5)^3$

*Quotient

$= \boxed{\log_6 \frac{(x+2)^4}{(x-5)^3}}$

Ex 12: $\underbrace{\log_2 5 + \log_2 x}_{\text{*Product}} - \log_2 3$

$= \log_2 (5x) - \log_2 3$

*Quotient

$= \boxed{\log_2 \left(\frac{5x}{3}\right)}$

Ex 13: $1 + 3\log_4 x$

*power

$= 1 + \log_4 x^3$

rewrite : $\log_b b = 1$

$= \log_4 4 + \log_4 x^3$

*Product property!

$= \log_4 (4x^3)$

If time....

Ex 14: $\log_4 4x^2$

Product

Expand!

$= \log_4 4 + \log_4 x^2$

$= \boxed{1 + 2\log_4 x}$

Ex 15: $\log_3 \sqrt{x-2}$

Expand!

$= \log_3 (x-2)^{1/2}$

$= \frac{1}{2}\log_3 (x-2)$

Ex 16: $\frac{3}{2}\log_9 x^6 - \frac{3}{4}\log_9 x^8$

Condense!!

$= \log_9 (x^6)^{3/2} - \log_9 (x^8)^{3/4}$

$= \log_9 x^9 - \log_9 x^6$

$= \log_9 \frac{x^9}{x^6} = \boxed{\log_9 x^3}$